Midterm

- Don't Panic. Keep calm and take it a step at a time.
- Write your student id *clearly* at the bottom of this page and on the top right on every odd page (Do this now).
- The midterm exam contains 5 problems. You have 90 minutes to earn 90 points.
- The midterm exam is closed book. You may bring one double-sided sheet of A4 paper to the midterm exam. (You may not bring any magnification equipment!) You may **not** use a calculator, your mobile phone, or any other electronic device.
- Write your solutions in the space provided. If you need more space, please use the scratch paper at the end of the midterm. Do not put part of the answer to one problem on a page for another problem.
- When writing your solutions, you can use the algorithms and data structures we have discussed in class without describing them in detail. Unless you make a modification, you do not have to describe how the standard methods work.
- Read through the problems before starting. Do not spend too much time on any one problem.
- Show your work. Partial credit will be given. You will be graded not only on the correctness of your answer, but also on the clarity with which you express it. Be neat.
- If you finish early, drop off your midterm in the front, and leave quietly.
- You may not bring any part of this midterm (even the scratch pages) out of the room.
- Good luck!

Problem #	**Name**	**Possible Points**	**Achieved Points**
1	True or False	15	
2	Herbert Goes to Space	10	
3	Algorithm Analysis	20	
4	Permutations	20	
5	Tree to Tape	25	
Total:		90	

Student id.: _____

IMPORTANT: Please ensure your student id is correct and legible.

Please circle your discussion group:

Si Jie 10am-12pm B110	Eldon 10am-12pm 0113	Shaowei 10am-12pm B111	Nicholas 12am-2pm B110	Advay 12am-2pm B111	Irham 2am-4pm 0120	David 4am-6pm B109	Enzio 4am-6pm B111	Jerrell 4am-6pm B108

Problem 1. True or False. [15 points]

Problem 1.a. Write (T)rue or (F)alse for the statements below about Mergesort.

☐ — Mergesort is *not* an example of a divide-and-conquer algorithm.

☐ — The merge step takes $O(n)$ time .

☐ — Replacing the merge step with Hoare's partitioning algorithm without modifying the remainder of the mergesort algorithm will still produce a valid sort.

☐ — The worst case computational complexity mergesort is $O(n^2)$.

☐ — Mergesort is in-place but is not stable.

Problem 1.b. Write (T)rue or (F)alse for the statements below about a (binary) **min-heap**.

☐ — The root is the largest element in the min-heap.

☐ — For every node in min-heap that has two children, the value of the node must be smaller than or equal to the value of its children.

☐ — When a heap is stored in an array, the array is sorted from smallest to largest.

☐ — Given a min-heap, you can output a sorted list of all the values in *decreasing* order in $O(n)$ time.

☐ — Given an unsorted array of values, you can build a heap in $O(n)$ time.

Problem 1.c. Write (T)rue or (F)alse for the statements below about AVL trees.

☐ — If key u is in an AVL tree, you can find the successor of u (or report no successor) in $O(\log n)$ time.

☐ — The left and right subtrees of an AVL tree are themselves height-balanced.

☐ — The rightmost element in an AVL tree is the smallest element.

☐ — Given an AVL tree, you can output a sorted list of all the keys in $O(n \log n)$ time using in-order traversal.

☐ — If a node in an AVL tree has two children, its predecessor is the rightmost node in its left subtree.

Problem 2. Herbert Goes to Space. [10 points]

Our favorite robotic clown Herbert wants to buy a trip on the GalaticX spaceship (designed by the famous Elan Mosk) to one of n possible planets. Distances to the n planets are stored in an array D such that $D[i]$ gives the distance of planet i from from Earth. Herbert wants to travel as far as possible, but doesn't have much money (he's lazy, remember?). Using all his SmileBucks savings, he can only afford a trip to the k-th closest planet. For this problem, assume all distances to be unique.

Problem 2.a. (4 points) **Describe the most time efficient algorithm you can think of to help Herbert find the k-th closest planet.** Be precise, but pseudocode or Java is not necessary unless it helps you to explain. You can assume you already have access to all the algorithms and data structures we have discussed in class. Unless you make a modification, you do not have to describe how the standard methods work. Write the time and space complexity of your method below:

Running time: Space:

Your algorithm with explanation:

The GalacticX space program has just increased the number of planets accessible to $n > 10^{12}$. This is much too large to fit into Herbert's spare memory, which can only store up to $O(k)$ elements. Instead, Herbert has access to the following functions:

- `long getNumPlanets()`: returns total number of planets accessible via GalacticX.
- `long getDistance(long i)`: returns the distance of the planet i from Earth.

Problem 2.b. (6 points) **Describe the most time-efficient algorithm you can think of to find the k-the closest planet, given that you can only use $O(k)$ space.** Be precise, but pseudocode or Java is not necessary unless it helps you to explain. You can assume you already have access to all the algorithms and data structures we have discussed in class. Unless you make a modification, you do not have to describe how the standard methods work. Write the time and space complexity of your method below:

Running time: Space:

Your algorithm with explanation:

Problem 3. Algorithm Analysis [20 points]
For each of the following, choose the best (tightest) asymptotic function T from among the given options. Some of the following may appear more than once, and some may appear not at all.

A. $O(1)$	B. $O(\log n)$	C. $O(n)$	D. $O(n \log n)$
E. $O(n^2)$	F. $O(n^3)$	G. $O(2^n)$	H. None of the above.

Problem 3.a.

$$T(n) = \frac{3n^7 - 4n - 17}{2n^5} \qquad T(n) =$$

Problem 3.b.

$$T(n) = 3n^2 + \sum_{i=1}^{n!} \frac{n}{5^i} \qquad T(n) =$$

Problem 3.c. The running time of the following code, as a function of n:

```
public static int calcj(int n){
    int j = 0;
    while (n > 0) {
        for (int i=0; i<n-1; i++) {
            j++;
        }
        n--;
    }
    return j;
}
```

$T(n) =$

Problem 3.d.

$$\begin{aligned} T(n) &= T\left(\frac{2n}{10}\right) + T\left(\frac{3n}{10}\right) + T\left(\frac{5n}{10}\right) + 2n \\ T(1) &= 1 \end{aligned}$$

$T(n) =$

Problem 4. Permutations [20 points]
For this problem, you are given an array $A = [0, 1, 2, 3, \ldots, n-1]$ of size n. Your goal is to generate all possible permutations of A. For example, given $A = [0, 1, 2]$ ($n = 3$), your program should output:

```
[0,1,2]
[0,2,1]
[1,0,2]
[1,2,0]
[2,0,1]
[2,1,0]
```

Your algorithm need **not** follow the exact sequence of permutations above, but it should print out each of the different permutations exactly once.

Problem 4.a. (8 points) **Describe a time-efficient algorithm for enumerating the all the permutations of A.** Be brief but precise. In addition to your explanation, **provide pseudocode or java code**. You can assume you already have access to all the algorithms and data structures we have discussed in class. Unless you make a modification, you do not have to describe how the standard methods work. State the time and space complexity of your method.

Running time: Space:

Explanation with Pseudocode/Java code:

Problem 4.b. (12 points) We can impose a total order on permutations using dictionary ordering. A permutation P is less than a permutation Q, denoted as $P < Q$, if in the first place i (starting from index 0) that P and Q differ, $P[i] < Q[i]$. As an example, `[0,1,2] < [0,2,1]` since $P[1] < Q[1]$. The smallest permutation under dictionary ordering is `[0,1,2]` and the largest is `[2,1,0]`.

Given a permutation P, find the *next largest* permutation under dictionary ordering. For example, given `[1,2,0]`, your program should output `[2,0,1]`. Describe the most time efficient algorithm you can think of. Be precise, but pseudocode or Java is not necessary unless it helps you to explain. What is the time and space efficiency of your algorithm?

Running time: Space:

Your algorithm with explanation:

Problem 5. Tree to Tape [25 points]

Naruto was exploring the office and found a big box of tape storage! Before the advent of hard disks and solid-state drives (SSDs), data used to be stored on magnetic tape. It might surprise you to learn that tape is still being used today; compared to other data storage technologies, tape is cheap and can last decades.

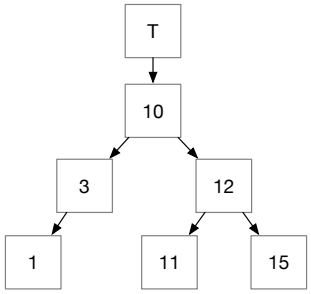
For this problem, we want to save binary search trees (BSTs) to tape for long-term storage. But one limitation of tape drives is their *sequential access* nature; sequential data transfer is fast, but random access is very slow. So, we want to convert a given BST to a *sorted doubly linked list*, which is more easily written to (and read from) the tape drive. Assume you have a standard BST with the following node structure:

Figure 1: A Magnetic Tape Drive. (Image Credit: Wikipedia)

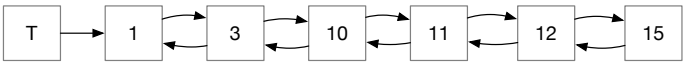
```
class Node {
    Node left
    Node right

    int key
    Object value
}
```

As a specific example, given a tree T:



Your method should create the following sorted doubly linked list:



You can reuse the nodes in the BST by using `left` as the `prev` reference and `right` as the `next` reference in the doubly linked list. Recall that in a doubly linked list, `prev` points to the previous node, and `next` points to the next node.

Problem 5.a. (10 points) **Describe an efficient method that converts a Binary Search Tree (BST) to a doubly-linked list.** Be brief but precise. For this problem, assume that the keys are *unique*. Explain the time and space complexity of your algorithm.

Running time: Space:

Your algorithm with explanation:

Now that we have managed to store our BST to tape, we need a way to convert the stored doubly linked list back to a BST. One trivial way might be just to use the linked list as a lop-sided BST. But this is inefficient since many operations will take $O(n)$ time. Instead, we want to convert the doubly linked list to a *height-balanced* BST.

Problem 5.b. (15 points) **What is a an efficient way to convert the data back from a doubly linked list to a *height-balanced* BST?** Provide a description for your algorithm. Be precise, but pseudocode or Java is not necessary unless it helps you to explain. What is the worst-case time and space complexity of your method in terms of the number of nodes n?

Hint: *It is possible to solve this problem in $O(n)$ time and use less than $O(n)$ space.*

Running time: Space:

Your algorithm with explanation:

Scratch Paper

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