True, False, Explain

Decide whether each of the following statements is true or false, and give a reason.

Problem 1. In a connected, weighted graph, every lowest weight edge is always in *some* minimum spanning tree.

[True / False]

Problem 2. For a connected, weighted graph with n vertices and exactly n edges, it is possible to find a minimum spanning tree in $O(n)$ time.

[True / False]

Problem 3. Negating all the edge weights in a weighted undirected graph G and then finding the minimum spanning tree gives us the maximum-weight spanning tree of the original graph G.

[True / False]

Problem 4. In a graph with unique edge weights, the spanning tree of second lowest weight is unique.

[True / False]

Problem 5. If a graph has a unique shortest path P from node s to node t, and has a unique minimum spanning tree T, then every edge in P must also be in T.

[True / False]

Problem 6. In a simple, undirected, connected, weighted graph with at least three vertices and unique edge weights, the heaviest edge in the graph is in no minimum spanning tree.

[True / False]

Problem 7. Suppose that T is a minimum spanning tree of G. If we increase the weight of each edge of G by the same positive amount δ, T is not guaranteed to be a minimum spanning tree.

[True / False]

Problem 8. An AVL tree is balanced, therefore a median of all elements in the tree is always at the root or one of its two children.

[True / False]

Problem 9. If every node in a binary search tree has either 0 or 2 children, then performing a find operation on the tree takes $O(\log n)$ time.

[True / False]

Problem 10. Let P be a shortest path from some vertex s to some other vertex t in a directed graph. If the weight of each edge in the graph is increased by one, P will still be a shortest path from s to t.

[True / False]

Problem 11. If a weighted directed graph G is known to have no shortest paths longer than k edges, then it suffices to run Bellman-Ford for only k passes in order to solve the single-source shortest paths problem on G.

[True / False]

Problem 12. BFS takes $O(V+E)$ time irrespective of whether the graph is presented with an adjacency list or with an adjacency matrix.

[True / False]