NATIONAL UNIVERSITY OF SINGAPORE

SCHOOL OF COMPUTING
SEMESTER II (2018-2019)

MOCK FINAL EXAM FOR
CS2040: DATA STRUCTURES AND ALGORITHMS

24 April 2019, 1pm
Time Allowed: 2h

MATRICULATION NUMBER:

A	0	1						

INSTRUCTIONS TO CANDIDATES

1. Write your matriculation number in the space provided above.
2. This examination paper consists of **10 MCQ questions and 2 structured questions** and comprises **ten (10)** printed pages including this front page.
3. Answer the MCQ questions by shading the correct option on the OCR form and the programming questions directly in the space given after each question. If necessary, ask for an extra sheet of paper and attach it at the end of the test
4. Marks allocated to each question are indicated. Total marks for the paper is 100.

EXAMINER'S USE ONLY			
Questions	**Possible**	**Marks**	**Check**
1 - 10	30		
11	15		
12	55		
Total	100		

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Some questions taken from materials used in CS2040 AY18/19 Sem 1.

Section 1. MCQ Questions (3 marks each)

1. Given a hash table of fixed size m, load factor α, hash function hash(*key*) = *key* mod m and collisions resolved using quadratic probing, for which values of m and α below will quadratic probing definitely terminate?

   A. $m = 15, \alpha = 0.3$
   B. $m = 15, \alpha = 0.5$
   C. $m = 17, \alpha = 0.3$
   D. $m = 17, \alpha = 0.5$
   E. $m = 17, \alpha = 0.7$

2. Which of the following statements are **always true** for a binary max-heap?

   I. The largest element is positioned at the rightmost leaf node.
   II. For every node with two children, the value of the left child must be greater than that of the right child.
   III. When a binary max-heap is implemented using an array, the array is sorted from largest to smallest.
   IV. Given a heap of n elements, you can output a sorted list of values in $O(n)$ time.

   A. IV only
   B. I and III only
   C. II and IV only
   D. III and IV only
   E. None of the above

3. Given a directed acyclic graph with n nodes where it is possible for exactly 2 nodes to reach all the other $n - 2$ nodes in the graph, what is the maximum possible number of topological orderings of the graph?

   A. $n - 2$
   B. n
   C. $(n - 2)!$
   D. $n!$
   E. None of the above

4. Given the following hash function hash(*key*) = (3 × *key*) mod 13, which of the following is the resulting hash table (of size 13) after the following sequence of operations if linear probing is used? The leftmost slot has index 0.

   add 5, add 1, add 18, add 14, add 3, add 10, del 14, add 12

   A.

		5	1	18	10	DEL			3	12		

   B.

		5	1	18	DEL	10				3	12	

   C.

		5	1	18	14	10			3	12		

   D.

		5	1	18	DEL	10			3	12		

   E.

		5	1	10	DEL	18			3	12		

5. What's the tightest Big-*O* time complexity for the following segment of code?

```
public void BigO(int n) {
     if (n == 0) return;
     int k = n;
     for (int i = 0; i < n; i++) {
          k += n;
          for (int j = 0; j < n; j++) {
               k--;
          }
     }
     BigO(k/2);
}
```

   A. O(log *n*)
   B. O(*n*)
   C. O(*n* log *n*)
   D. O(n^2)
   E. O(n^2 log *n*)

6. Which of the following statements is **true**?

   A. Given an AVL tree, you can output a sorted list of all the keys in O(n) time using pre-order traversal.
   B. An AVL tree can be converted into a max heap in O(n) time.
   C. The balance factor of any node in a balanced AVL tree must be between 0 and 1.
   D. At most log n rotations are required after inserting a node into an AVL tree.
   E. The predecessor of any node in an AVL tree, if exists, can be found in O(1) time.

7. Given a graph with V vertices and E edges stored in an adjacency matrix, what is the tightest time complexity of DFS, Bellman Ford and Floyd Warshall respectively?

   A. O(V+E), O(VE), O(V^3)
   B. O(V+E), O(E^2), O(V^3)
   C. O(V^2), O(VE), O(V^3)
   D. O(V^2), O(E^2), O(V^3)
   E. None of the above

8. Which of the following statements is **true**?

   A. In a connected graph with N vertices and N edges, relaxing all the edges in only one iteration of Bellman Ford is sufficient to detect a negative cycle.
   B. In a directed, connected graph with N vertices and N edges, a topological ordering of the vertices definitely exists.
   C. There exists a directed complete graph with more than 3 vertices such that a valid topological ordering exists.
   D. If there are negatively weighted edges in a graph, then it is impossible for any variant of Dijkstra's Algorithm to solve the Single Source Shortest Path problem.
   E. None of the above.

9. To heapify an array of 8 elements, what is the maximum number of comparisons needed?

   A. 8
   B. 9
   C. 10
   D. 11
   E. 12

10. Which of the following statements are **true**?

   I. Quicksort is an in-place sorting algorithm
   II. Selection Sort is a stable sorting algorithm
   III. Insertion Sort can be simulated using one queue.
   IV. Selection Sort can be simulated using one queue.

   A. I and III only
   B. II and III only
   C. II and IV only
   D. I, II and IV only
   E. I, III and IV only

Section 2: Analysis Questions (15 marks)

Prove (the statement is correct) or disprove (the statement is wrong) the statements below.

11a. Suppose we have a graph with n vertices and m edges. First run the Bellman-Ford Algorithm on the graph from source s. Then, relax all edges for a further n^2 iterations. Only all vertices v where dist(s, v) < 0 (according to the algorithm above) have $-\infty$ shortest distance. (3 marks)

11b. Suppose we have a undirected graph with non-negative edge weights. If we run the Modified Dijkstra's Algorithm (mentioned in lecture) with a max Priority Queue, we cannot solve the Single-Source Shortest Path problem. (3 marks)

11c. The following hash function is **not** a good hash function for strings. (3 marks)

```
int hash(String s) {
    int a = 11, h = 0;
    for (int i = 0; i < s.length(); i++) {
        h %= a;
        h *= ((int)s.charAt(i) + 11);
        a += 2;
    }
    return h + a;
}
```

11d. In each iteration of radix sort, we order the data into groups according to the next character in each data. Insertion sort can be used to perform this ordering. (3 marks)

11e. In a max binary heap data structure, we are able to find the successor of any element in constant time. (3 marks)

Section 3: Application Questions (55 marks)

12a) In a city of *n* towns, *m* directed roads connect two towns. Each road has a positive height limit, where only vehicles with height lower than the height limit can travel on and pass through the road. Design an algorithm that allows you to find, for every pair of towns, the height of the highest vehicle that can travel between them. (5 marks)
Hint: The Floyd Warshall's Algorithm taught in class is provided for you below.

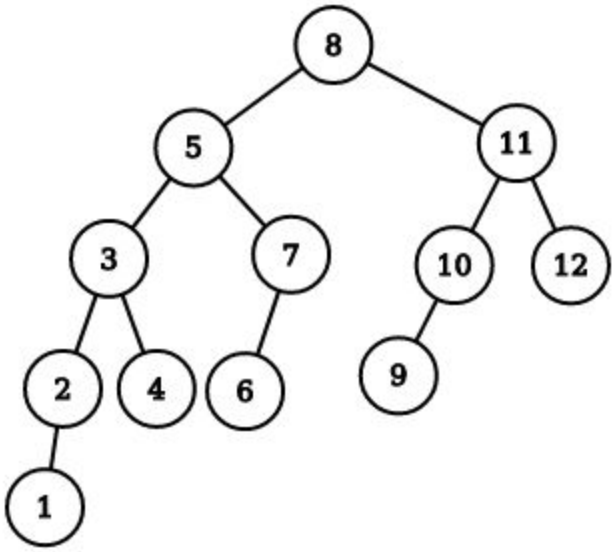
```
Let the number of nodes in the graph be V.
Let D be an adjacency matrix.
Let D[i][i] = 0, D[i][j] = the weight of edge(i, j) if there is an
edge from i to j, otherwise ∞.

for (int k = 0; k < V; k++)
    for (int i = 0; i < V; i++)
        for (int j = 0; j < V; j++)
            D[i][j] = Math.min(D[i][j], D[i][k] + D[k][j]);
```

12b) In a city of *n* towns, *m* directed roads connect two towns. There are two types of roads: normal roads and toll roads. Coco's home is located at node 1 and he wants to get to school, located at node *n*. Describe the most efficient algorithm you can think of to help him find the minimum number of toll roads he has to pass through on his way from home to school. (10 marks)

12c) In a city of *n* towns, *m* directed roads connect two towns. Roads are of positive lengths, some are long, some are short, and two roads may be of the same length. Coco's home is located at node 1 and he wants to get to school, located at node *n*, via a simple path. The *boringness* of a path is defined as the *k*th longest road on that path. Describe an algorithm to find the *boringness* of the least boring path among all paths Coco can take. (15 marks)

12d) Given the following AVL tree, show what happens when node 11 is deleted. (10 marks)



12e) Given a sorted array of *n* integers, write a method `buildAVL` that builds a height balanced AVL tree in O(*n*). (15 marks)

End of Paper