NATIONAL UNIVERSITY OF SINGAPORE

CS2040 – DATA STRUCTURES AND ALGORITHMS

(Semester 4: AY2018/19)

Time Allowed: 2 Hours

INSTRUCTIONS TO STUDENTS

1. Do **NOT** open the question paper until you are told to do so.
2. This assessment paper consists of **Twelve (12)** printed pages and **Nine (9)** questions with possible subsections.

   Important tips: Pace yourself! Do **not** spend too much time on one (hard) question.
3. Answer all questions. Write your answer for the structured questions directly in the space given after each question.
4. You are allowed to use PENCIL to write your answers in this question paper.
5. When this Assessment starts, **please immediately write your Student Number** below (using pen). No extra time given at the end of the exam to do so!
6. This is a **Open Book Assessment**.

STUDENT NUMBER:

A								

(Write your Student Number above legibly with a pen.)

Questions	Possible	Marks
Q1-5	15	
Q6	15	
Q7	20	
Q8	20	
Q9	30	
Total	100	

Section A – Analysis (15 Marks)

Prove (the statement is correct) or disprove (the statement is wrong) the following statements below. If you want to prove it, provide the proof or at least a convincing argument. If you want to disprove it, provide at least one counter example. 3 marks per each statement below (1 mark for circling true or false, 2 marks for explanation):

1. With the partition method of quicksort as written below, the time complexity of the partition method is O(N) in the worst case where $N = j - i + 1$. **[true/false]**

```
public static int partition(int[] a, int i, int j) {
  int p = a[i]; // p is the pivot, the i-th item
  int m = i;    // Initially S1 and S2 are empty
  for (int k=i+1; k<=j; k++) {
    if (a[k] < p) {
      m++;
      for (int c=k; c > m; c--) {
        int temp = a[c];
        a[c] = a[c-1];
        a[c-1] = temp;
      }
    }
  }
  int temp = a[i];
  a[i] = a[m];
  a[m] = temp;
  return m;
}
```

2. When an item at index i of a min-heap is updated with a new value, calling both ShiftUp(i) and ShiftDown(i) will re-heapify the min-heap. However only one of the operations will cause swaps to happen or none will cause any swap to happen (item remains where it is). It is impossible for both to cause swaps to take place. **[true/false]**

3. The smallest bipartite graph in terms of number of edges is a tree. **[true/false]**

4. For the counting component algorithm, since each vertex is visited once and all its neighbor are processed, the time complexity can be written as the sum of all neighbors processed for all vertices, which is O(E) in general instead of O($V + E$) or O(max(V, E)). **[true/false]**

5. For any connected weighted undirected graph, Prim's algorithm will always maintain a sub-tree of the MST as the algorithm progresses until the final MST is obtained at the end of the algorithm. Kruskal's algorithm on the other hand will always produce a forest of subtrees of the MST (disconnected subtrees of the MST) at least once during the running of the algorithm, but will merge these subtrees into the final MST at the end. You may assume for Prim's for edges with same weight, they are ordered by neighbor vertex number, and for Kruskal's the edge list is sorted first by edge weight then by 1st vertex number then 2nd vertex number. **[true/false]**

Section B – Applications (85 Marks)

Write in pseudo-code (code-like or English description is ok as long as it is clear and specific). Any algorithm/data structure/data structure operation not taught in CS2040 must be described, there must be no black boxes. Partial marks will be awarded for correct answers not meeting the time complexity required.

6. **Another linked list manipulation question … [15 marks]**

Given a BasicLinkedList A where each node contains a string, write the algorithm for the method

public BasicLinkedList NewList(BasicLinkedList A, int p)

which will create a new BasicLinkedList that only contains strings of length >= p from A in the same relative ordering. This may result in an empty list returned if there are no strings in A which are as long or longer than p. You cannot destroy A in the process of creating the new linkedlist. You cannot use any of the methods in the BasicLinkedList class and the Java LinkedList API. You have access to the head of A. This means you have to make use of references to ListNode and manipulate next pointers to achieve your goal. You can create new ListNode if required, and have access to operations in ListNode.

7. **Can we make it simpler … [20 marks]**

a) John works in a diamond processing factory. His friend Tom has made a machine that will scan each diamond that passes by the machine on a conveyor belt. The scan will first tag the diamond with a unique id number and also determine the weight of the diamond. This pair of info will then be inserted into a min-heap with the weight as key. After every 1 hr, the smallest diamond is obtained by looking at the root of the min-heap and thrown away. The min-heap is then reset and the whole process is repeated. Even though using a min-heap result in a O(lgN) per insertion where N is the # of diamonds scanned per hr and peeking at the top of the heap is O(1) time, John cannot help but think this solution is too complicated for what it is trying to achieve and can be made simpler and more efficient. Please design an even simpler algorithm without using min-heap that will do the same thing but is even more efficient (i.e better time complexity)! **[10 marks]**

b) Jerry works in the same diamond factory, and he deals with cutting the diamonds after the unwanted ones have been thrown away. The diamonds will come to him in batches of size N where 100 <= N <= 1,000,000. Now if the diamonds are too small or too large (based on weight), they require special processing which is not handled by Jerry. Thus he needs to efficiently select only those which are within a certain weight range $[X, Y]$ where X <= weight <= Y. Tom has yet again suggested building another machine that will again scan the diamonds one by one and insert them by their weight into an AVL tree. The algorithm is then as follows
1.) Build the AVL tree in O($NlgN$) time.
2.) Search for diamond p with weight just >= X → O(lgN)
3.) From p just keep calling successor until diamond with weight <= Y → O(N)
4.) Return all diamonds found in 2.) as the diamonds to be processed → O(N)
Total time complexity is O($NlgN$)

Jerry just like John thinks this is too complicated for what he is trying to achieve, and has asked you to once again design an even simpler algorithm without using AVL, that will do the same thing but is even more efficient! **[10 marks]**

8. **Can it be done without …. [20 marks]**

Given a graph G which is a subgraph of another connected graph G', we want to test if G is also a spanning tree of G'. Give an algorithm to do this without using BFS/DFS or any of their variants. You are only given the number of vertices V in G' and the edge-list E representing G.

9. **Star Track [30 marks]**

a) Captain Pica has led his team of intrepid galactic explorers to planet XIV. On this planet is located an underground network of junctions and connecting tunnels (Pica has a map of this underground network). There are N junctions, including the starting junction where Pica and his team is and at each junction is planted a photon bomb. If all N of them explode then the planet will be destroyed. These junctions are connected to each other via tunnels that can be travelled in both directions and there are $N-1$ to $\frac{1}{2}N^2$ such tunnels. There is always a way to reach any junction from any other junction via tunnels, and all the tunnel are different in length.

In order to stop the photon bombs from going off, Pica and his team (there are also N members in the team including Pica) start off as a group at the starting junction. At any newly discovered junction including the starting junction, 1 member will stay behind to disarm the bomb (it takes a long time to disarm a bomb) and the rest can split up into smaller groups if necessary to travel 1 to k of the k unexplored tunnels connected to the junction.

Since there are N members there must be a way from the starting junction for all members to each reach a junction and disarm the photon bomb there. Model the underground network of junctions and tunnels as a graph, and give an algorithm to minimize the total distances of tunnels travelled (count the distance travelled by a group along a tunnel as simply the length of the tunnel) and each junction is reached by 1 member of Pica's team. **[18 marks]**

i.) What do the vertices and edges in your graph represent? Explain. **[2 marks]**

ii.) Are the edges directed or undirected? Explain. **[2 marks]**

iii.) Is this a connected or disconnected graph? Explain. **[2 marks]**

iv.) What are the weights in your graph? Explain. **[2 marks]**

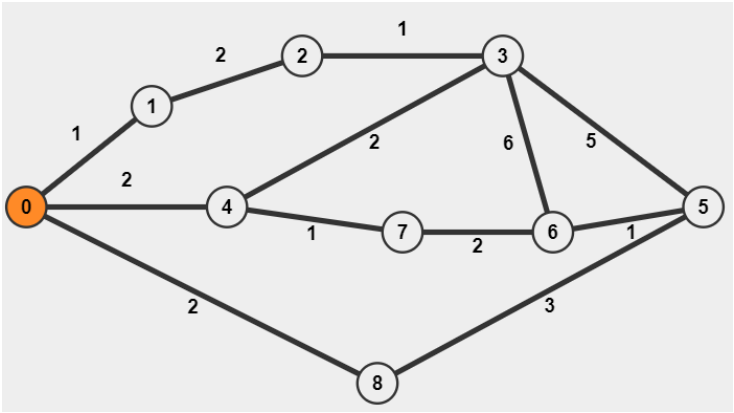
v.) Using the graph as described above give the most efficient algorithm you can think of to minimize total distances of tunnel travelled and also allow each junction to be reached by 1 member of Pica's team. **[10 marks]**

b) After saving planet XIV, Pica and his team have returned to their space ship. They now need to travel to M ($1 <= M <= 100$) planets to disarm all the photon bombs there, and the M planets must be visited in a specific sequence $\{A_1, A_2, \ldots, A_m\}$ that is given. The navigator of Pica's space ship uses a star map to travel the galaxy. This map is basically a graph that models discovered planets as vertices and there will be a bi-directed edge linking 2 planets if the straight line path between them have been explored previously (edge weight is distance between the planets). The graph is not a complete graph since not all straight line path between all pairs of planets have been explored, but it will be a connected graph meaning there is at least one path between any pair of discovered planets. There are between 1,000 and 100,000 discovered planets, and between 10,000 and 1,000,000 edges connecting planets.

To move from some planet a to some other planet b, the navigator will always use the **shortest distance** to get from a to b from the star map. However, from the starting point (which is planet XIV), once the space ship has reached some planet c from planet XIV by travelling P distance, the space ship can warp from c to XIV immediately and from XIV the space ship can warp back and forth between XIV and any planet that has shortest distance <= P from planet XIV (imagine P is the "radius" of warp from planet XIV), thus effectively making shortest distance between XIV and such planets **ZERO**. However once the spaceship warps, it uses up all its fuel and if it does not land back at planet XIV or any of the M planets to refill, the space ship will be marooned in space. Whenever the spaceship travels a greater distance than the current P from planet XIV, P will be updated to this new distance. At the start, P = 0.

Given the above scenario, give an algorithm so that the navigator will minimize the travel distance to all M planets in the sequence required, starting from planet XIV.

E.g If there are 4 planets to be visited and the visitation sequence is {2,4,3,5} (numbering is their vertex number in the star map), and the star map is as follows (vertex 0 is planet XIV).



Bi-directed edge represented as undirected for ease of viewing

First the space ship will start at vertex 0 (planet XIV), and visit 2,4,3,5 as follows

- **Vertex 2** – Since P is currently 0, we have to travel to 2 via the path 0-1-2 with shortest distance 3. Now P = 3.
- **Vertex 4** – From vertex 2, the space ship can warp back to vertex 0 and then warp to vertex 4 for a total of 0 distance since shortest distance from vertex 0 to vertex 4 is only 2 which is < P = 3. Now P is still 3
- **Vertex 3** – From vertex 4 we can travel the edge 4-3 to reach vertex 3 with cost 2, but this is not optimal, instead warp back to vertex 0, then warp to vertex 2 and travel the edge 2-3 for a cost of 1. Now P = 3+1 = 4.
- **Vertex 5** – From vertex 3 we can travel the edge 3-5 to get to vertex 5 with cost 5 but again this is not optimal. We can possibly warp back to vertex 0 then warp to vertex 7 and use the edges 7-6-5, since shortest distance from vertex 0 to vertex 7 is only 3 and currently P = 4. However, since 7 is not in the visitation sequence, the space ship has run out of fuel and cannot move. The optimal is to warp back to vertex 0 from vertex 3 then warp to vertex 4 and travel the path 4-7-6-5 for a total cost of 4. Now P = 4+4 = 8.

We have now visited 2,4,3,5 in sequence and used a minimized total distance of 8.

Given the star map (a connected undirected weighted graph G(V,E)), and the visitation sequence $\{A_1,\ldots A_M\}$ of M planets, start from vertex 0 (planet XIV) and describe the most efficient algorithm you can think of that will visit all planets in the sequence given and travel the shortest distance possible. **[12 marks]**

~~~ END OF PAPER ~~~